Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Higher level
Paper 1

14 May 2026

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

Session number

2 hours

Instructions to students

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

14 pages

2226 – 5025

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show **all** working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

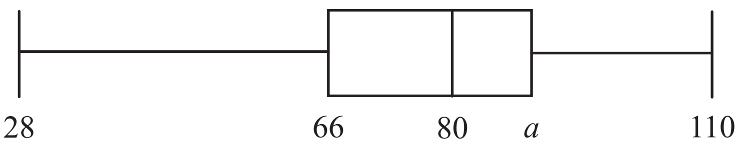
A school has 600 students across four grade levels as shown in the following table.

Grade	**Number of students**
9	114
10	179
11	137
12	170

A group of 60 students was surveyed using a stratified sampling technique, such that each grade in the sample was proportional to the population. Each student was asked how much allowance, in Singapore dollars ($), they received per week.

(a) Find the number of Grade 12 students in the sample. [2]

The results of the survey are summarized in the following box and whisker diagram.



Survey of allowances from 60 students

(b) State the median. [1]

The interquartile range is $24.

(c) Write down the value of a. [1]

An outlier is a value that is less than $Q_1 - 1.5 \times \text{IQR}$ or greater than $Q_3 + 1.5 \times \text{IQR}$.

(d) Determine the smallest possible allowance, from the sample, that is not an outlier. [2]

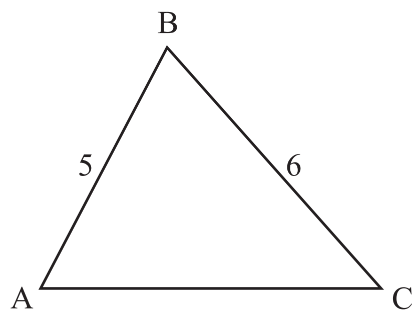
(This question continues on the following page)

(Question 1 continued)

2. [Maximum mark: 7]

The following diagram shows triangle ABC. It is known that $\mathrm{AB} = 5\,\mathrm{cm}$, $\mathrm{BC} = 6\,\mathrm{cm}$, and $\cos \mathrm{A\hat{B}C} = \frac{1}{5}$ where $\mathrm{A\hat{B}C}$ is acute.

diagram not to scale



(a) Find AC. [3]

(b) (i) Show that $\sin \mathrm{A\hat{B}C} = \frac{2\sqrt{6}}{5}$.

(ii) Hence, find the area of triangle ABC. [4]

3. [Maximum mark: 6]

In this question, p and r are positive integers greater than 1.

(a) Given that $\log_3 p = 3 + \log_3 r$, show that $p = 27r$. [2]

(b) Hence, find the value of r and the value of p that satisfy these simultaneous equations:

$$\log_3 p = 3 + \log_3 r$$

$$\log_r p = 4$$

[4]

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4. [Maximum mark: 5]

Find the exact value of $\sec 75°$. Express your answer in the form $\sqrt{m}+\sqrt{n}$, where m , $n \in \mathbb{Z}^{+}$.

5. [Maximum mark: 5]

Let $y = x^4 - \sin 4x$.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [2]

(b) Hence, or otherwise, find $\int (x^4 - \sin 4x)^7 (x^3 - \cos 4x)\,\mathrm{d}x$. [3]

6. [Maximum mark: 6]

The function f is defined by $f(x) = 4 - \frac{7}{3x+2}$, where $x \in \mathbb{R}$, $x \neq -\frac{2}{3}$.

(a) Find an expression for $f^{-1}(x)$. [3]

The function g is defined such that $(f \circ g)\left(\frac{x}{3}\right) = x$.

(b) Using your answer to part (a) or otherwise, find an expression for $g(x)$. [3]

7. [Maximum mark: 6]

Consider an infinite convergent geometric sequence, with first three terms u_1, u_2, u_3.

The sum of the first k terms of this sequence is equal to half of the sum of the infinite geometric sequence.

It is given that $u_{k+1} = \frac{1}{16}$.

(a) Show that $u_1 = \frac{1}{8}$. [4]

(b) Find u_{2k+1}. [2]

8. [Maximum mark: 6]

Consider the complex number

$$z = 3 + k\mathrm{i},$$

where $k \in \mathbb{R}$, $k < 0$, and $-\pi < \arg(z) \leq \pi$.

Given that z^5 is a real number, find the values of k in the form $a \tan b\pi$, where a, $b \in \mathbb{R}$, $b < 0$.

9. [Maximum mark: 8]

Prove by mathematical induction that $\frac{1}{n+1}+\frac{1}{n+2}+\frac{1}{n+3}+\cdots+\frac{1}{2n}\geq\frac{7}{12}$, for $n\in\mathbb{Z}^{+}$, $n\geq 2$.

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 16]

The functions f and g are defined for $0 \leq x \leq \pi$ by

$$f(x) = \sin 2x$$
$$g(x) = 1 - \cos 2x\,.$$

(a) Solve the equation $f(x) = g(x)$ for $0 \leq x \leq \pi$. [6]

(b) Sketch the graphs of $y = f(x)$ and $y = g(x)$ on the same set of axes, clearly labelling the points of intersection with their coordinates. [4]

The graphs of f and g intersect at $x = a$ and $x = b$, where $0 < a < b$.

(c) Find the area of the region enclosed by the graphs of f and g for $a \leq x \leq b$. [6]

Do **not** write solutions on this page.

11. [Maximum mark: 20]

The equation of the plane Π has the form $2x + y + az = b$, where a and b are real constants.

The line L_1 is parallel to Π but does not lie on Π.

L_1 passes through the point $\mathrm{A}(7, -1, 1)$ and has direction vector $\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$.

(a) (i) Show that $a = -2$.

(ii) Deduce the range of possible values of b. [4]

It is then given that $b = 2$.

The line L_2 passes through the point A and has direction vector $\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$.

L_2 intersects Π at the point N.

(b) Find the coordinates of N. [5]

The acute angle between L_2 and Π is α.

(c) Find the exact value of $\sin\alpha$. [4]

(d) Given that $\mathrm{AN} = \dfrac{9\sqrt{3}}{5}$, find the distance between L_1 and Π. [2]

(e) Find the vector equation of the line formed when L_1 is reflected in Π. [5]

Do **not** write solutions on this page.

12. [Maximum mark: 19]

Consider $y = \tan\left(1 - e^{3x}\right)$.

(a) Show that $\dfrac{dy}{dx} = -3e^{3x}\left(1 + y^2\right)$. [4]

(b) Show that $\dfrac{d^2y}{dx^2} = -9$, when $x = 0$. [5]

The function f is defined by $f(x) = \tan\left(1 - e^{3x}\right)$, where $-0.2 \le x \le 0.2$.

(c) Find the Maclaurin series for $f(x)$, up to and including the term in x^2. [2]

(d) Hence, or otherwise, find $\lim\limits_{x \to 0} \dfrac{\tan\left(1 - e^{3x}\right)}{x}$. [3]

The answer to part (c) is equal to the first two non-zero terms in the series expansion of $\dfrac{x}{a + bx}$, where a, $b \in \mathbb{Q}$.

(e) Find the value of a and the value of b. [5]

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Answers written on this page
will not be marked.

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