

© International Baccalaureate Organization 2026

All rights reserved. No part of this product may be reproduced in any form or by any electronic or mechanical means, including information storage and retrieval systems, without the prior written permission from the IB. Additionally, the license tied with this product prohibits use of any selected files or extracts from this product. Use by third parties, including but not limited to publishers, private teachers, tutoring or study services, preparatory schools, vendors operating curriculum mapping services or teacher resource digital platforms and app developers, whether fee-covered or not, is prohibited and is a criminal offense.

More information on how to request written permission in the form of a license can be obtained from <https://ibo.org/become-an-ib-school/ib-publishing/licensing/applying-for-a-license/>.

© Organisation du Baccalauréat International 2026

Tous droits réservés. Aucune partie de ce produit ne peut être reproduite sous quelque forme ni par quelque moyen que ce soit, électronique ou mécanique, y compris des systèmes de stockage et de récupération d'informations, sans l'autorisation écrite préalable de l'IB. De plus, la licence associée à ce produit interdit toute utilisation de tout fichier ou extrait sélectionné dans ce produit. L'utilisation par des tiers, y compris, sans toutefois s'y limiter, des éditeurs, des professeurs particuliers, des services de tutorat ou d'aide aux études, des établissements de préparation à l'enseignement supérieur, des fournisseurs de services de planification des programmes d'études, des gestionnaires de plateformes pédagogiques en ligne, et des développeurs d'applications, moyennant paiement ou non, est interdite et constitue une infraction pénale.

Pour plus d'informations sur la procédure à suivre pour obtenir une autorisation écrite sous la forme d'une licence, rendez-vous à l'adresse <https://ibo.org/become-an-ib-school/ib-publishing/licensing/applying-for-a-license/>.

© Organización del Bachillerato Internacional, 2026

Todos los derechos reservados. No se podrá reproducir ninguna parte de este producto de ninguna forma ni por ningún medio electrónico o mecánico, incluidos los sistemas de almacenamiento y recuperación de información, sin la previa autorización por escrito del IB. Además, la licencia vinculada a este producto prohíbe el uso de todo archivo o fragmento seleccionado de este producto. El uso por parte de terceros —lo que incluye, a título enunciativo, editoriales, profesores particulares, servicios de apoyo académico o ayuda para el estudio, colegios preparatorios, desarrolladores de aplicaciones y entidades que presten servicios de planificación curricular u ofrezcan recursos para docentes mediante plataformas digitales—, ya sea incluido en tasas o no, está prohibido y constituye un delito.

En este enlace encontrará más información sobre cómo solicitar una autorización por escrito en forma de licencia: <https://ibo.org/become-an-ib-school/ib-publishing/licensing/applying-for-a-license/>.

# Mathematics: analysis and approaches

## Higher level

### Paper 1

14 May 2026

Zone A afternoon | Zone B afternoon | Zone C afternoon

Session number

2 hours

|  |  |  |  |  |  |  |  |  |  |
|--|--|--|--|--|--|--|--|--|--|
|  |  |  |  |  |  |  |  |  |  |
|--|--|--|--|--|--|--|--|--|--|

#### Instructions to students

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.



Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

## Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

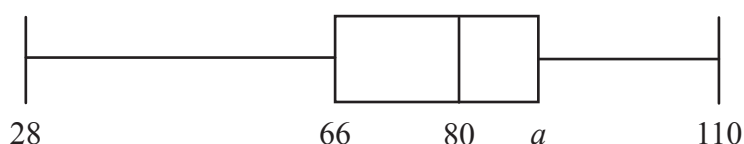
A school has 600 students across four grade levels as shown in the following table.

| Grade | Number of students |
|-------|--------------------|
| 9     | 114                |
| 10    | 179                |
| 11    | 137                |
| 12    | 170                |

A group of 60 students was surveyed using a stratified sampling technique, such that each grade in the sample was proportional to the population. Each student was asked how much allowance, in Singapore dollars (\$), they received per week.

(a) Find the number of Grade 12 students in the sample. [2]

The results of the survey are summarized in the following box and whisker diagram.



Survey of allowances from 60 students

(b) State the median. [1]

The interquartile range is \$24.

(c) Write down the value of  $a$ . [1]

An outlier is a value that is less than  $Q_1 - 1.5 \times \text{IQR}$  or greater than  $Q_3 + 1.5 \times \text{IQR}$ .

(d) Determine the smallest possible allowance, from the sample, that is not an outlier. [2]

(This question continues on the following page)



(Question 1 continued)

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



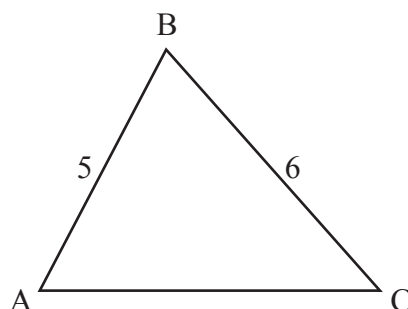
16EP03

Turn over

**2.** [Maximum mark: 7]

The following diagram shows triangle  $ABC$ . It is known that  $AB = 5$  cm,  $BC = 6$  cm, and  $\cos \hat{ABC} = \frac{1}{5}$  where  $\hat{ABC}$  is acute.

**diagram not to scale**



- (a) Find AC. [3]

- (b) (i) Show that  $\sin \hat{ABC} = \frac{2\sqrt{6}}{5}$ .

- (ii) Hence, find the area of triangle ABC. [4]

[illegible]

3. [Maximum mark: 6]

In this question,  $p$  and  $r$  are positive integers greater than 1.

(a) Given that  $\log_3 p = 3 + \log_3 r$ , show that  $p = 27r$ . [2]

(b) Hence, find the value of  $r$  and the value of  $p$  that satisfy these simultaneous equations:

$$\log_3 p = 3 + \log_3 r$$

$$\log_r p = 4$$
 [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



4. [Maximum mark: 5]

Find the exact value of  $\sec 75^\circ$ . Express your answer in the form  $\sqrt{m} + \sqrt{n}$ , where  $m, n \in \mathbb{Z}^+$ .

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



5. [Maximum mark: 5]

Let  $y = x^4 - \sin 4x$ .

(a) Find  $\frac{dy}{dx}$ . [2]

(b) Hence, or otherwise, find  $\int (x^4 - \sin 4x)^7 (x^3 - \cos 4x) dx$ . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



6. [Maximum mark: 6]

The function  $f$  is defined by  $f(x) = 4 - \frac{7}{3x+2}$ , where  $x \in \mathbb{R}$ ,  $x \neq -\frac{2}{3}$ .

(a) Find an expression for  $f^{-1}(x)$ . [3]

The function  $g$  is defined such that  $(f \circ g)\left(\frac{x}{3}\right) = x$ .

(b) Using your answer to part (a) or otherwise, find an expression for  $g(x)$ . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



7. [Maximum mark: 6]

Consider an infinite convergent geometric sequence, with first three terms  $u_1, u_2, u_3$ .

The sum of the first  $k$  terms of this sequence is equal to half of the sum of the infinite geometric sequence.

It is given that  $u_{k+1} = \frac{1}{16}$ .

(a) Show that  $u_1 = \frac{1}{8}$ . [4]

(b) Find  $u_{2k+1}$ . [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



8. [Maximum mark: 6]

Consider the complex number

$$z = 3 + ki,$$

where  $k \in \mathbb{R}$ ,  $k < 0$ , and  $-\pi < \arg(z) \leq \pi$ .

Given that  $z^5$  is a real number, find the values of  $k$  in the form  $a \tan b\pi$ , where  $a, b \in \mathbb{R}$ ,  $b < 0$ .

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



9. [Maximum mark: 8]

Prove by mathematical induction that  $\frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \cdots + \frac{1}{2n} \geq \frac{7}{12}$ , for  $n \in \mathbb{Z}^+$ ,  $n \geq 2$ .

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



Do **not** write solutions on this page.

### Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

**10.** [Maximum mark: 16]

The functions  $f$  and  $g$  are defined for  $0 \leq x \leq \pi$  by

$$\begin{aligned} f(x) &= \sin 2x \\ g(x) &= 1 - \cos 2x. \end{aligned}$$

(a) Solve the equation  $f(x) = g(x)$  for  $0 \leq x \leq \pi$ . [6]

(b) Sketch the graphs of  $y = f(x)$  and  $y = g(x)$  on the same set of axes, clearly labelling the points of intersection with their coordinates. [4]

The graphs of  $f$  and  $g$  intersect at  $x = a$  and  $x = b$ , where  $0 < a < b$ .

(c) Find the area of the region enclosed by the graphs of  $f$  and  $g$  for  $a \leq x \leq b$ . [6]



Do **not** write solutions on this page.

11. [Maximum mark: 20]

The equation of the plane  $\Pi$  has the form  $2x + y + az = b$ , where  $a$  and  $b$  are real constants.

The line  $L_1$  is parallel to  $\Pi$  but does not lie on  $\Pi$ .

$L_1$  passes through the point  $A(7, -1, 1)$  and has direction vector  $\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$ .

(a) (i) Show that  $a = -2$ .

(ii) Deduce the range of possible values of  $b$ . [4]

It is then given that  $b = 2$ .

The line  $L_2$  passes through the point  $A$  and has direction vector  $\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ .

$L_2$  intersects  $\Pi$  at the point  $N$ .

(b) Find the coordinates of  $N$ . [5]

The acute angle between  $L_2$  and  $\Pi$  is  $\alpha$ .

(c) Find the exact value of  $\sin \alpha$ . [4]

(d) Given that  $AN = \frac{9\sqrt{3}}{5}$ , find the distance between  $L_1$  and  $\Pi$ . [2]

(e) Find the vector equation of the line formed when  $L_1$  is reflected in  $\Pi$ . [5]



Do **not** write solutions on this page.

12. [Maximum mark: 19]

Consider  $y = \tan(1 - e^{3x})$ .

(a) Show that  $\frac{dy}{dx} = -3e^{3x}(1 + y^2)$ . [4]

(b) Show that  $\frac{d^2y}{dx^2} = -9$ , when  $x = 0$ . [5]

The function  $f$  is defined by  $f(x) = \tan(1 - e^{3x})$ , where  $-0.2 \leq x \leq 0.2$ .

(c) Find the Maclaurin series for  $f(x)$ , up to and including the term in  $x^2$ . [2]

(d) Hence, or otherwise, find  $\lim_{x \rightarrow 0} \frac{\tan(1 - e^{3x})}{x}$ . [3]

The answer to part (c) is equal to the first two non-zero terms in the series expansion of  $\frac{x}{a + bx}$ , where  $a, b \in \mathbb{Q}$ .

(e) Find the value of  $a$  and the value of  $b$ . [5]



Please **do not** write on this page.

Answers written on this page  
will not be marked.



16EP15

Please **do not** write on this page.

Answers written on this page  
will not be marked.



16EP16